Your Signature _____

1. (10 points) Find the general solution (with appropriate justification) to the differential equation given by
    $$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 13x = 0,$$
    for $t > 0$.

2. (12 points) Munuram, a college student going to AMCE owes Rs 10,000 to UCO bank which charges an annual rate of 10%. Munuram makes payments continuously at a constant rate of Rs 100 per month (i.e. Rs 1200 per year). Set up and solve the initial value for Munuram's loan. Find the time T that it will take to pay off the debt.

3. (13 points) Consider the Bernoulli differential equation given by

    $$\frac{dy}{dt} = y(9 - y^2),$$

    for $t > 0$.

    (a) Suppose $y(0) = \alpha > 0$. Find the solution to the above initial value problem.

    (b) Suppose $y(0) = 0$. Find the solution to the above initial value problem.

    (c) Find the $\lim_{t \to \infty} y(t)$ in (a) and (b).

4. (15 points) Consider the differential equation given by

    $$\frac{d^2x}{dt^2} + p(t)\frac{dx}{dt} + q(t)x = 0,$$

    for $t \geq 0$, and p, q are continuous functions.

    (a) Suppose $y_1(t) = 15(t + e)^2$ and $y_2(t) = -4(t + e)^3 \ln(t + e)$ are two possible solutions of the above ODE. Find the Wronksian of y_1 and y_2. Can you determine $p(t)$ for each $t > 0$?

    (b) Suppose $q(t) = 0$ and $p(t) < -2$ for all $t \geq 0$. Let x be a non-trival solution to the above ODE. Find the cardinality of Z where

    $$Z = \{a \in [0, \infty) : x(a) = 0\}.$$